Randlay Primary School and Nursery

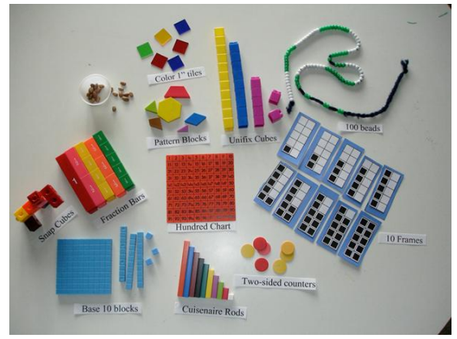
KS1 Multiplication and Division

Concrete/Pictorial/Abstract

Maths Calculation Policy

This policy has created using a wide range of external sources (NCETM, White Rose Maths Hub and Click4teaching). It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

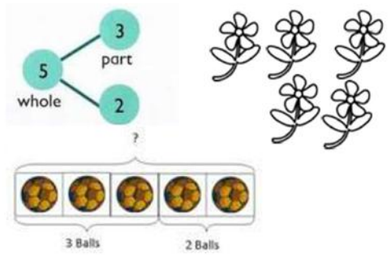
Concrete



The first part of the CPA approach is Concrete, which is known as the 'doing' stage. children use concrete objects to model problems. The CPA approach brings concepts to life by allowing children to experience and handle physical (concrete) objects, unlike traditional maths teaching methods where teachers demonstrate how to solve a problem. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

As part of the CPA approach, new concepts are introduced through the use of physical objects or practical equipment. These can be physically handled, enabling children to explore different mathematical concepts. These are sometimes referred to as maths manipulatives and can include ordinary household items such as straws or dice, or specific mathematical resources such as dienes or numicon.

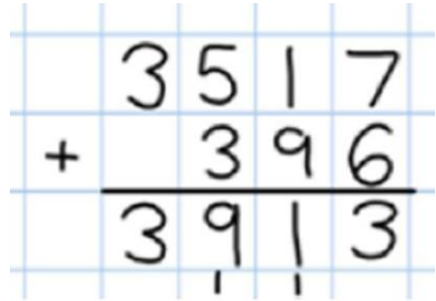
Pictorial



The second part of the CPA approach is pictorial, which is known as the 'seeing' stage. This is when visual representations of concrete objects are used to model problems. children are encouraged to make a mental connection between the physical object they have just handled and the abstract pictures, diagrams or models that represent the objects from the problem.

Building or drawing a model makes it easier for children to grasp difficult abstract concepts (for example, fractions). Once children are confident with a concept using concrete resources, they progress to drawing pictorial representations or quick sketches of the objects. By doing this, they are no longer manipulating the physical resources, but still benefit from the visual support the resources provide.

Abstract



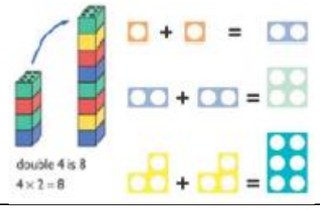
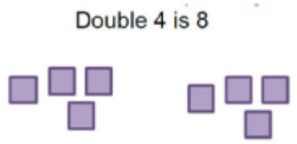
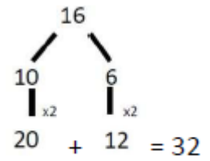
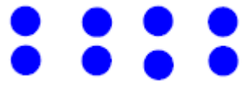
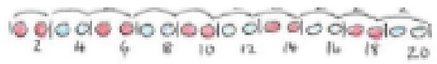
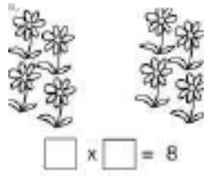
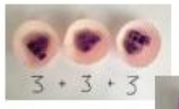
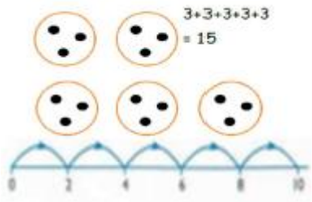
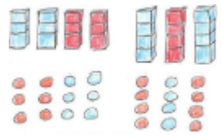
The final stage of the CPA approach is abstract, which is known as the 'symbolic' stage, where children use abstract symbols to model problems. children will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols).

Once children have a secure understanding of the concept through the use of concrete resources and visual images, they are then able to move on to the abstract stage. Here, children are using abstract symbols to model problems – usually numerals. To be able to access this stage effectively, children need access to the previous two stages alongside it.

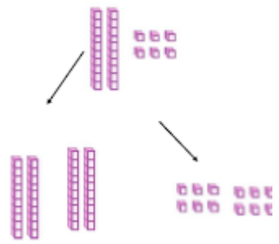
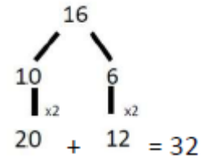
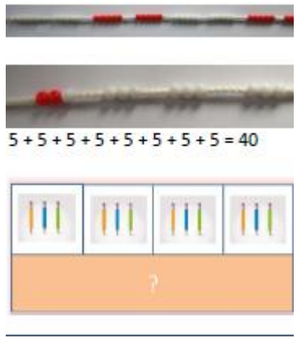
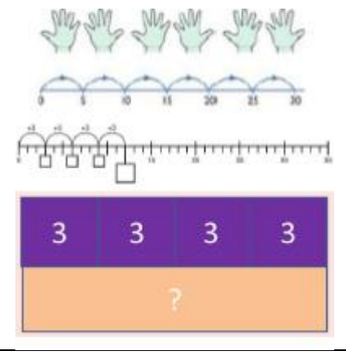
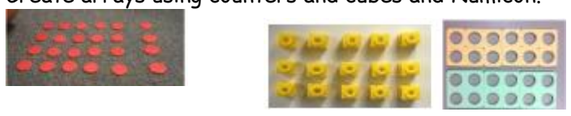
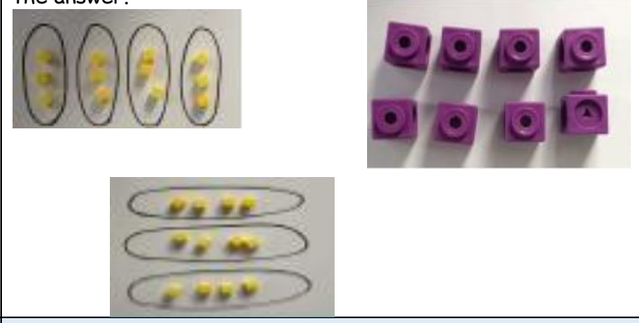
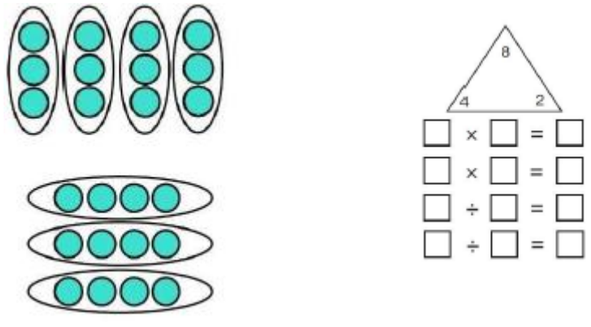
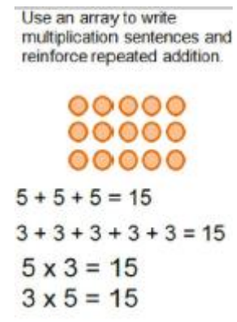
Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task. Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by the maths subject leader. Using this will support children with their examinations and throughout secondary school.

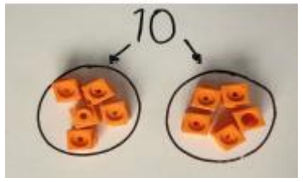
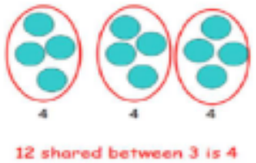
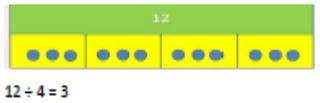
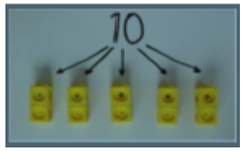
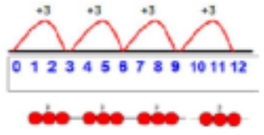
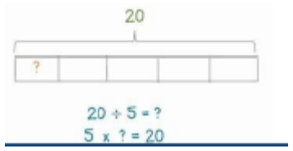
Year 1 Multiplication

	Vocabulary: count in ones, twos… tens… array, groups of, equal groups odd, even		**Instructional vocabulary:** carry on, continue repeat what comes next? find, choose, collect use, make, build, tell me, describe, pick out, talk about, explain, show me, read, write, record
Objective/Strategy	**Concrete "doing stage"**	**Pictorial "seeing stage"**	**Abstract "symbolic stage"**
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s)	 Count the groups as children are skip counting, children may use their fingers as they are skip counting.	 Children make representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	  Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8
Repeated addition	   Use different objects to add equal groups	Use pictorial including number lines to solve There are 3 sweets in one bag How many sweets are in 5 bags altogether 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10
Known facts	Count in multiples of twos, fives and tens.		
Essential Knowledge by the end of Year 2.	Count in 2s	Doubles up to 10	
	Count in 10s	Double multiples of 10	
	Count in 5s	Count in 2s, 5s and 10s	

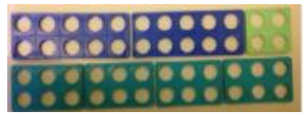
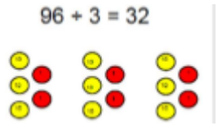
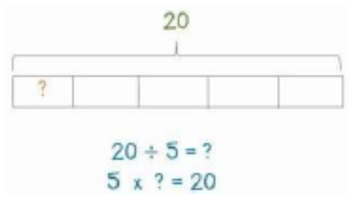
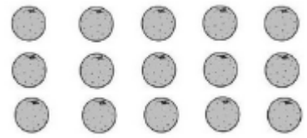
Year 2 Multiplication

	Vocabulary: lots of, groups of ×, times, multiply, multiplied by multiple of once, twice, three times… ten times… times as (big, long, wide… and so on) repeated addition array row, column double, halve share, share equally		**Instructional vocabulary:** carry on, continue, repeat, what comes next? predict describe the pattern describe the rule, find, find all, find different, investigate
Objective/Strategy	**Concrete "doing stage"**	**Pictorial "seeing stage"**	**Abstract "symbolic stage"**
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. 	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$
Multiplication is commutative Create arrays using Using the Inverse This should be taught alongside division, so pupils learn how they work alongside each other.	Create arrays using counters and cubes and Numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer. 	Use representations of arrays to show different calculations and explore commutativity. 	12 = 3 × 4 12 = 4 ×3 2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences. 
Known facts	Recall and use x and ÷ facts for the 2, 5 and 10 x tables, including recognising odd and even numbers.		
Essential Knowledge by the end of Year 2.	2 x table	Doubles up to 20	
	10 x table	Doubles of multiples of 5	
	5x table	Count in 3s	

Year 1 Division

	Vocabulary: count in ones, twos… tens… share, groups of, equal groups odd, even		**Instructional vocabulary:** count out, share out, left, left over
Objective/Strategy	**Concrete "doing stage"**	**Pictorial "seeing stage"**	**Abstract "symbolic stage"**
Division as sharing Use Gordon ITPs for modelling	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  Sharing:  Children use bar modelling to show and support understanding. 	12 shared between 3 is 4
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping    Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Known facts	Count in multiples of twos, fives and tens.		
Essential Knowledge by the end of Year 1.	Count back in 2s	Halves up to 10	
	Count back in 10s	Halve multiples of 10	
	Count back in 5s	How many 2s? 5s? 10s?	
Tests of divisibility	All even numbers will divide by 2		

Year 2 Division

	Vocabulary: share, share equally one each, two each, three each… group in pairs, threes… tens equal groups of ÷, divide, divided by, divided into left, left over		**Instructional vocabulary:** tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of… show how you
Objective/Strategy	**Concrete "doing stage"**	**Pictorial "seeing stage"**	**Abstract "symbolic stage"**
Division as grouping Use cubes, counters, objects or place value counters	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Known facts	Recall and use x and ÷ facts for the 2, 5 and 10 x tables, including recognising odd and even numbers.		
Essential Knowledge by the end of Year 2.	Division facts (2 x table)	Halves up to 20	
	Division facts (10 x table)	Review division facts (2 x, 5 x, 10 x tables)	
	Division facts (5 x table)	Count back in 3s	
Tests of divisibility	All numbers ending in 0 will divide by 10	All numbers ending in 5 and 0 will divide by 5	